

Laws of Differentiation



Rules of Differentiation

Scalar Product of function $\frac{d}{dx}[af(x)] = af'(x)$
Linear Combination of functions <small>[For Sum and Difference of Functions]</small> $\frac{d}{dx}[af(x) \pm bg(x)] = af'(x) \pm bg'(x)$
Product Rule: <small>[For Product of Functions]</small> $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
Quotient Rule: <small>[For Rational Functions]</small> $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
Chain Rule: <small>[For Composite Functions]</small> $\frac{d}{dx}[f(g(x))] = f'[g(x)] \cdot g'(x)$
$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

Implicit Differentiation

Function that cannot express y in terms of x

$$\frac{d}{dx}f(y) = \frac{d}{dx}\left(\frac{dy}{dy}\right)f(y) = \frac{dy}{dx}\left(\frac{d}{dy}\right)f(y) = \frac{dy}{dx}f'(y)$$

Logarithmic Differentiation

Function that where base and power is variable

$$y = [f(x)]^{f(x)}$$

$$\log_a y = f(x) \log_a [f(x)]$$

Implicit differentiation

Parametric Differentiation

A function express in terms of 3rd variables

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

No	Type	General Functions	Gradient Functions
		y	$\frac{dy}{dx}$
1	Algebraic	a (constant term)	0
		$[f(x)]^n$	$n[f(x)]^{n-1}f'(x)$
2	Exponential	$e^{f(x)}$	$[f'(x)]e^{f(x)}$
		$a^{f(x)}$	$[f'(x)]a^{f(x)}[\ln a]$
3	Logarithmic	$\ln[f(x)]$	$\frac{f'(x)}{f(x)}$
		$\log_a[f(x)]$	$\frac{f'(x)}{[f(x)][\ln a]}$
4	Trigonometric	$\sin[f(x)]$	$f'(x)\cos[f(x)]$
	Inverse Trigonometric	$\cos[f(x)]$	$-f'(x)\sin[f(x)]$
		$\tan[f(x)]$	$f'(x)\sec^2[f(x)]$
MF26		$\frac{d}{dx}\sec x = \sec x \tan x$	$f'(x)\sec[f(x)]\tan[f(x)]$
		$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$	$-f'(x)\operatorname{cosec}[f(x)]\cot[f(x)]$
		$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$	$-f'(x)\operatorname{cosec}^2[f(x)]$
		$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}, x < 1$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, f(x) < 1$
		$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}, x < 1$	$-\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, f(x) < 1$
		$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$	$\frac{f'(x)}{1+[f(x)]^2}$

Radian Measure:

Differentiation holds for x measured in radians.

If x never depicts as x⁰, assume radian. If x is in degree, convert x degrees to radian.

Inverse Presentation:

Usually $\sin^n x = (\sin x)^n$ for all integers n except n = -1.

By convention, $(\sin x)^{-1}$ is written as cosec x while $\sin^{-1}x$ is the **inverse function** of sin x.

Similarly, $(\cos x)^{-1} = \sec x$ and $(\tan x)^{-1} = \cot x$

$$\pi \text{ radians} = 180^\circ$$

$$\frac{\pi}{180} \text{ radians} = 1^\circ$$

$$\frac{\pi x}{180} = x^\circ$$

Further Derivatives of Parametric Function

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{d^{n-1}y}{dx^{n-1}}\right]}{\left(\frac{dx}{dt}\right)}$$

Trigonometry Formula

Complementary Angles	Pythagorean Identities	Negative Angles
$\sin(90^\circ - \theta) = \cos \theta$	$\sin^2 \theta + \cos^2 \theta = 1$	$\sin(-\theta) = -\sin \theta$
$\cos(90^\circ - \theta) = \sin \theta$	$1 + \tan^2 \theta = \sec^2 \theta$	$\cos(-\theta) = \cos \theta$
$\tan(90^\circ - \theta) = \frac{1}{\tan \theta}$	$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$	$\tan(-\theta) = -\tan \theta$

Reciprocal Functions Compound Angle Identities

$\tan \theta = \frac{\sin \theta}{\cos \theta}; \cot \theta = \frac{1}{\tan \theta}$	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B$
$\sec \theta = \frac{1}{\cos \theta}; \operatorname{cosec} \theta = \frac{1}{\sin \theta}$	$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	

Multiple Angle Identities

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Factor Formula

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Laws of Logarithmic Functions

Given a complicated Logarithmic functions, simplify using

Power Law : $k \log_a x = \log_a x^k$

Product Law : $\log_a xy = \log_a x + \log_a y$

Quotient Law : $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Change of Base Law: $\log_a b = \frac{\log_c b}{\log_c a}$